

# Photon energy absorption rate of a striped Hall gas

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Using symmetries of the current correlation function, we analyze the frequency dependence of the photon energy absorption rate of a striped Hall gas. Since the magnetic translational symmetry is spontaneously broken in the striped Hall gas, a Nambu-Goldstone (NG) mode appears. It is shown that the NG mode causes a sharp absorption at the zero energy in the long wavelength limit by using the single mode approximation. The photon energy absorption rate at the NG mode frequency strongly depends on the direction of the wave number vector. Whereas, the absorption rate at the cyclotron frequency does not depend on the direction of the wave number vector in the long wavelength limit. The cyclotron resonance is not affected in the striped Hall gas. Our result supplements the Kohn's theorem in the system of the NG mode.

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## I. INTRODUCTION

The two-dimensional electron system under a strong magnetic field, the quantum Hall system (QHS), without impurities has four symmetries, that is the electromagnetic  $U(1)$ , the magnetic rotation, and the magnetic translations in  $x$  and  $y$  direction. The magnetic translational symmetry in one direction and the magnetic rotational symmetry are spontaneously broken in a striped Hall gas, which is supposed to be realized at the half-filling of the third and higher Landau Level (LL)s. The striped Hall gas has an anisotropic Fermi surface, and leads to a highly anisotropic resistivity.<sup>1,2</sup> Since the magnetic translation is spontaneously broken in this state, a Nambu-Goldstone (NG) mode appears. Dynamical effects of the NG mode are studied in the present paper.

There are several theoretical works which would lead to the highly anisotropic resistivity at the half-filled third and higher LLs. The Hartree-Fock approximation (HFA) at the half-filled  $l$ th LL predicts two solutions, a unidirectional charge density wave (UCDW)<sup>3,4,5,6</sup> and a highly anisotropic charge density wave (ACDW).<sup>7</sup> The striped Hall gas is the UCDW which has an energy gap in one direction and no energy gap in another direction. The ACDW has an energy gap in each direction. Collective modes for the UCDW have been studied based on the edge current picture,<sup>8,9,10,11,12</sup> the generalized random phase approximation (GRPA),<sup>13</sup> and the single mode approximation (SMA).<sup>14,15,16</sup> Collective modes for the ACDW have been studied in the time-dependent HFA.<sup>7</sup> We use the UCDW and the SMA to study the photon absorption effect of the NG mode.

The Kohn's theorem<sup>17</sup> is a quite general theorem concerning the photon absorption in the QHS. It includes two statements on the cyclotron resonance. The first one is that a sharp absorption of a homogeneous rotating microwave occurs only at the cyclotron frequency. The second one is that the cyclotron resonance is not affected by electron interactions. The implicit assumption for the first statement is that the zero-energy excited state is absent in the long wavelength limit. The assumption is not satisfied in the striped Hall gas.

It is an open question if a sharp absorption at the NG mode frequency occurs in the long wavelength limit. A photon energy absorption rate is proportional to the imaginary part of a current correlation function, and is inversely proportional to the frequency  $\omega$  of the electromagnetic vector potential  $\mathbf{A}_{ext} = \mathcal{E}e^{i\mathbf{k}\cdot\mathbf{r}+ik_zz-i\omega t}/i\omega$ . Here  $\mathcal{E}$  is the polarization vector,  $(\mathbf{k}, k_z)$  is the wave number vector and  $(\mathbf{r}, z, t)$  is the space-time coordinate. We project the system onto the  $l$ th LL and study the contribution of the NG mode. Since the imaginary part of the LL projected current correlation function becomes zero in the  $|\mathbf{k}| \rightarrow 0$  limit, it would appear that the photon energy absorption rate is zero. However, when  $\omega$  equals the NG mode frequency  $\omega_{NG}(\mathbf{k})$ , the denominator also becomes zero in the  $|\mathbf{k}| \rightarrow 0$  limit. Hence, the  $|\mathbf{k}| \rightarrow 0$  limit of the photon energy absorption rate could be finite. Actually, we find that a sharp absorption occurs at the NG mode frequency. The photon energy absorption rate at the NG mode frequency strongly depends on the direction of  $\mathbf{k}$ .

Without the LL projection, we study the cyclotron resonance in the striped Hall gas. The photon energy absorption rate at the cyclotron frequency does not depend on the direction of  $\mathbf{k}$  in the  $|\mathbf{k}| \rightarrow 0$  limit. Then we find that the cyclotron resonance is unaffected in the striped Hall gas phase. The second statement of the Kohn's theorem is intact.

This paper is organized as follows. In Sec. II, the symmetries of the QHS are clarified and the striped Hall gas is constructed as an eigenstate of charges of unbroken symmetries. The current correlation function is computed by means of the magnetic translational invariance of the striped Hall gas in Sec. III. A trigonometric factor of the momentum and the periodic Dirac's delta function of wave number vectors appear due to the stripe periodicity. In order to investigate the current correlation function in the long wavelength limit, we also use the Hartree-Fock (HF)

solution of the striped Hall gas in Sec. III. We discuss the photon energy absorption rate of the striped Hall gas and the f-sum rule by using the SMA spectrum in Sec. IV and give a discussion and summary in Sec. V. Appendix A gives a definition of the photon energy absorption rate. The cyclotron resonance is derived in appendix B.

## II. BROKEN SYMMETRIES OF THE STRIPED HALL GAS

The striped Hall gas spontaneously breaks the magnetic translational symmetry of the QHS. We clarify symmetries of the QHS, and define the striped Hall gas state in terms of conserved charges of the magnetic translational symmetry.

### A. Symmetries of the QHS

In the QHS, the Hamiltonian has four symmetries, that is the electromagnetic  $U(1)$  symmetry, the magnetic translational symmetries in  $x$  and  $y$  direction, and the magnetic rotational symmetry. First, we introduce conserved currents and charges for these symmetries. The two-dimensional vector potential  $A_i(\mathbf{r})$  is introduced by means of  $B = \partial_x A_y - \partial_y A_x$ . We ignore the spin degree of freedom and use the natural unit ( $\hbar = c = 1$ ) in the present paper. We introduce relative coordinates and guiding-center coordinates of the electron cyclotron motion. The relative coordinates are defined by  $\xi = (-i\partial_y + eA_y)/eB$  and  $\eta = -(-i\partial_x + eA_x)/eB$ . The guiding-center coordinates are defined by  $X = x - \xi$  and  $Y = y - \eta$ . These coordinates satisfy the following commutation relations,  $[X, Y] = -[\xi, \eta] = i/eB$  and  $[X, \xi] = [X, \eta] = [Y, \xi] = [Y, \eta] = 0$ . The operators  $X$  and  $Y$  are the generators of the magnetic translations of the one-electron state in  $-y$  direction and  $x$  direction, respectively. The total Hamiltonian  $H$  of the QHS is the sum of the free Hamiltonian  $H_0$  and the Coulomb interaction Hamiltonian  $H_{int}$  as follows,

$$\begin{aligned} H &= H_0 + H_{int}, \\ H_0 &= \int d^2 r \Psi^\dagger(\mathbf{r}) \frac{m\omega_c^2}{2} (\xi^2 + \eta^2) \Psi(\mathbf{r}), \\ H_{int} &= \frac{1}{2} \int d^2 r d^2 r' \Psi^\dagger(\mathbf{r}) \Psi^\dagger(\mathbf{r}') V(\mathbf{r} - \mathbf{r}') \Psi(\mathbf{r}') \Psi(\mathbf{r}), \end{aligned} \quad (2.1)$$

where  $\Psi(\mathbf{r})$  is the electron field operator,  $\omega_c = eB/m$ , and  $V(\mathbf{r}) = q^2/r$  ( $q^2 = e^2/4\pi\epsilon$ ,  $\epsilon$  is the background dielectric constant). We do not consider impurities and a confining potential in this paper. Noether currents of the electromagnetic  $U(1)$ , the magnetic translations in  $x$  and  $-y$  directions are defined by  $j^\mu = \Psi^\dagger v^\mu \Psi$ ,  $j_Y^\mu = \Psi^\dagger v^\mu Y \Psi + \delta_x^\mu \mathcal{L}/eB$  and  $j_X^\mu = \Psi^\dagger v^\mu X \Psi - \delta_y^\mu \mathcal{L}/eB$ . Here,  $v^\mu = (1, \mathbf{v})$ ,  $\mathbf{v} = \omega_c(-\eta, \xi)$  and  $\mathcal{L}$  is the Lagrangian density for the total Hamiltonian  $H$ . Conserved charges  $Q, Q_Y, Q_X$  for the electromagnetic  $U(1)$  and the magnetic translations are defined respectively by  $Q = \int d^2 r j^0(\mathbf{r})$ ,  $Q_Y = \int d^2 r j_Y^0(\mathbf{r})$  and  $Q_X = \int d^2 r j_X^0(\mathbf{r})$ . These charges commute with the total Hamiltonian  $H$  and obey  $[Q_X, Q_Y] = iQ/eB$ . The electromagnetic  $U(1)$  charge  $Q$  commutes with all conserved charges. We assume that  $Q$  is not broken and the ground state is the eigenstate of  $Q$  as  $Q|0\rangle = N_e|0\rangle$ , where  $N_e$  is the number of electrons.

### B. Symmetries of the striped Hall gas

The conserved charges of magnetic translations satisfy the following algebra,

$$\begin{aligned} \frac{\partial}{\partial x} j^\mu(\mathbf{r}) &= -\frac{2\pi}{ia^2} [j^\mu(\mathbf{r}), Q_Y], \\ \frac{\partial}{\partial y} j^\mu(\mathbf{r}) &= \frac{2\pi}{ia^2} [j^\mu(\mathbf{r}), Q_X], \end{aligned} \quad (2.2)$$

where  $a = \sqrt{2\pi/eB}$ . Those states which are periodic in one direction and uniform in another direction break the magnetic translational symmetry. Since an expectation value of the left hand side of Eq. (2.2) with respect to the states becomes nonzero, the states cannot be eigenstates of the conserved charge  $Q_X$  or  $Q_Y$ . Therefore the magnetic translational symmetry is spontaneously broken.

We construct the striped Hall gas state which breaks the magnetic translational symmetry in  $x$  direction and conserves the magnetic translational symmetry in  $y$  direction. The magnetic translations in  $x$  and  $y$  directions are generated respectively by  $\exp(i\frac{2\pi}{a^2}xQ_Y)$  and  $\exp(-i\frac{2\pi}{a^2}yQ_X)$  which is derived from Eq. (2.2). Hence, the striped Hall gas state is an eigenstate of an infinitesimal magnetic translation in  $y$  direction,  $\exp(-i\frac{2\pi}{a^2}yQ_X)$ , and is also an eigenstate of a stripe period magnetic translation in  $x$  direction,  $\exp(i\frac{2\pi}{a^2}r_s a n_x Q_Y)$ , where  $n_x$  is an integer and  $r_s a$  is

a period of the stripe. The striped Hall gas state is characterized by an eigenvalue  $\mathbf{K}^{(m)}$  of the magnetic translation generator and a species index  $\sigma$  as

$$\begin{aligned} e^{i\frac{2\pi}{a^2}r_s a n_x Q_Y} |\mathbf{K}^{(m)}, \sigma\rangle &= e^{ir_s a n_x K_x^{(m)}} |\mathbf{K}^{(m)}, \sigma\rangle, \\ e^{-i\frac{2\pi}{a^2}y Q_X} |\mathbf{K}^{(m)}, \sigma\rangle &= e^{-iy K_y^{(m)}} |\mathbf{K}^{(m)}, \sigma\rangle, \\ H |\mathbf{K}^{(m)}, \sigma\rangle &= E_\sigma(\mathbf{K}^{(m)}) |\mathbf{Q}_X^{(m)}, \sigma\rangle. \end{aligned} \quad (2.3)$$

The ground state obeys  $H|\mathbf{0}, \sigma_0\rangle = E_{\sigma_0}(\mathbf{0})|\mathbf{0}, \sigma_0\rangle$ . We assume a periodic boundary condition for the striped Hall gas in a rectangle with the length  $L_x a r_s$  and  $L_y a / r_s$  as

$$\begin{aligned} e^{i\frac{2\pi}{a^2}Q_Y L_x r_s a} |\mathbf{K}^{(m)}, \sigma\rangle &= |\mathbf{K}^{(m)}, \sigma\rangle, \\ e^{-i\frac{2\pi}{a^2}Q_X L_y \frac{a}{r_s}} |\mathbf{K}^{(m)}, \sigma\rangle &= |\mathbf{K}^{(m)}, \sigma\rangle, \\ \mathbf{K}^{(m)} = (K_x^{(m)}, K_y^{(m)}) &= \left( \frac{2\pi m_x}{L_x r_s a}, \frac{2\pi r_s m_y}{a L_y} \right), \end{aligned} \quad (2.4)$$

where  $L_x, L_y$  are integers,  $m_x = 0, \dots, L_x - 1$  and  $m_y = -[N_e L_y / 2], \dots, [N_e L_y / 2] - 1$  ([x] is the integral part of x). The thermodynamic limit  $L_x, L_y, N_e \rightarrow \infty$  is taken in the following. In this limit, the completeness becomes

$$\begin{aligned} \sum_{m,\sigma} |\mathbf{K}^{(m)}, \sigma\rangle \langle \mathbf{K}^{(m)}, \sigma| &= \sum_{\mathbf{m}, \sigma} \left| \frac{2\pi m_x}{L_x r_s a}, \frac{2\pi r_s m_y}{a L_y}, \sigma \right\rangle \langle \frac{2\pi m_x}{L_x r_s a}, \frac{2\pi r_s m_y}{a L_y}, \sigma | \\ &= \sum_{\sigma} \frac{\mathcal{A}}{(2\pi)^2} \int_0^{2\pi/(r_s a)} dK_x \int_{-\infty}^{\infty} dK_y |\mathbf{K}, \sigma\rangle \langle \mathbf{K}, \sigma| = 1, \end{aligned} \quad (2.5)$$

where  $\mathcal{A} = L_x L_y a^2$ .

### III. CURRENT CORRELATION FUNCTION

First, we represent a current correlation function in terms of the magnetic translational property of the striped Hall gas. Next, the current correlation function is projected onto the  $l$ th LL, and is evaluated by including the NG mode. We use the HFA and the SMA to evaluate the current correlation function.

#### A. Symmetry of the current correlation function

We set  $a = 1$  in the following calculation for the simplicity. The current correlation function of the striped Hall gas  $|\mathbf{0}, \sigma_0\rangle$  is defined by

$$\tilde{\Pi}^{\mu\nu}(\mathbf{k}, \mathbf{k}'; \omega, \omega') = \int dt dt' d^2r d^2r' \langle \mathbf{0}, \sigma_0 | T j^\mu(\mathbf{r}, t) j^\nu(\mathbf{r}', t') | \mathbf{0}, \sigma_0 \rangle e^{i\mathbf{k}\cdot\mathbf{r} + i\mathbf{k}'\cdot\mathbf{r}' - i\omega t - i\omega' t'}, \quad (3.1)$$

where  $T$  means the time-ordered product. Let us write the current operator as  $j^\mu(\mathbf{r}, t) = e^{iHt} j^\mu(\mathbf{r}, 0) e^{-iHt}$ . By using the completeness of energy eigenstates Eq. (2.5) between current operators in Eq. (3.1), the current correlation function becomes

$$\tilde{\Pi}^{\mu\nu}(\mathbf{k}, \mathbf{k}'; \omega, \omega') = -i2\pi\delta(\omega + \omega') K_\omega^{\mu\nu}(\mathbf{k}, \mathbf{k}'), \quad (3.2)$$

where

$$\begin{aligned} K_\omega^{\mu\nu}(\mathbf{k}, \mathbf{k}') &= \\ \sum_{\sigma} \frac{\mathcal{A}}{(2\pi)^2} \int_0^{2\pi/r_s} dK_x \int_{-\infty}^{\infty} dK_y &\left\{ \frac{\langle 0, \sigma_0 | \tilde{j}^\mu(\mathbf{k}, 0) | \mathbf{K}, \sigma \rangle \langle \mathbf{K}, \sigma | \tilde{j}^\nu(\mathbf{k}', 0) | 0, \sigma_0 \rangle}{\omega + E_\sigma(\mathbf{K}) - E_{\sigma_0}(\mathbf{0}) - i\delta} - \frac{\langle 0, \sigma_0 | \tilde{j}^\nu(\mathbf{k}', 0) | \mathbf{K}, \sigma \rangle \langle \mathbf{K}, \sigma | \tilde{j}^\mu(\mathbf{k}, 0) | 0, \sigma_0 \rangle}{\omega - E_\sigma(\mathbf{K}) + E_{\sigma_0}(\mathbf{0}) + i\delta} \right\} \end{aligned} \quad (3.3)$$

and  $\delta$  in the denominator is an infinitesimal positive constant.

Next, we transform  $K_\omega^{\mu\nu}(\mathbf{k}, \mathbf{k}')$  with the use of Eq. (2.2). The current operator is transformed under the magnetic translation as  $j^\mu(\mathbf{r}, 0) = e^{-i2\pi r_s n_x Q_Y} j^\mu(\bar{x}, y, 0) e^{i2\pi r_s n_x Q_Y}$  and  $j^\mu(\bar{x}, y, 0) = e^{i2\pi y Q_X} j^\mu(\bar{x}, 0, 0) e^{-i2\pi y Q_X}$ , where  $\mathbf{r} = (r_s n_x + \bar{x})\mathbf{e}_x + y\mathbf{e}_y$  with  $-r_s/2 \leq \bar{x} \leq r_s/2$ .  $\mathbf{e}_x$  and  $\mathbf{e}_y$  are unit vectors in  $x$  and  $y$  direction, respectively. Inserting these current operators into Eq. (3.3), we find that  $Q_X$  and  $Q_Y$  are replaced by eigenvalues of excited states as follows,

$$\begin{aligned}
K_\omega^{\mu\nu}(\mathbf{k}, \mathbf{k}') &= \sum_{\sigma} \frac{\mathcal{A}}{(2\pi)^2} \int_0^{2\pi/r_s} dK_x \int_{-\infty}^{\infty} dK_y \sum_{n_x, n'_x} \int_{-r_s/2}^{r_s/2} d\bar{x} d\bar{x}' \int_{-\infty}^{\infty} dy dy' e^{ik_x(r_s n_x + \bar{x}) + ik_y y + ik'_x(r_s n'_x + \bar{x}') + ik'_y y'} \\
&\quad \times \left\{ \frac{\langle \mathbf{0}, \sigma_0 | j^\mu(\bar{x}, 0, 0) | \mathbf{K}, \sigma \rangle \langle \mathbf{K}, \sigma | j^\nu(\bar{x}', 0, 0) | \mathbf{0}, \sigma_0 \rangle}{\omega + E_{\sigma}(\mathbf{K}) - E_{\sigma_0}(\mathbf{0}) - i\delta} e^{ir_s K_x(n_x - n'_x) + iK_y(y - y')} \right. \\
&\quad \left. - \frac{\langle \mathbf{0}, \sigma_0 | j^\nu(\bar{x}', 0, 0) | \mathbf{K}, \sigma \rangle \langle \mathbf{K}, \sigma | j^\mu(\bar{x}, 0, 0) | \mathbf{0}, \sigma_0 \rangle}{\omega - E_{\sigma}(\mathbf{K}) + E_{\sigma_0}(\mathbf{0}) + i\delta} e^{-ir_s K_x(n_x - n'_x) - iK_y(y - y')} \right\} \\
&= \sum_{\sigma} \frac{\mathcal{A}}{(2\pi)^2} \int_0^{2\pi/r_s} dK_x \int_{-\infty}^{\infty} dK_y \int_{-r_s/2}^{r_s/2} d\bar{x} d\bar{x}' e^{ik_x \bar{x} + ik'_x \bar{x}'} \sum_{n''_x} e^{i(k_x + k'_x)r_s n''_x} \int_{-\infty}^{\infty} dy' e^{i(k_y + k'_y)y'} \\
&\quad \times \left\{ \frac{\langle \mathbf{0}, \sigma_0 | j^\mu(\bar{x}, 0, 0) | \mathbf{K}, \sigma \rangle \langle \mathbf{K}, \sigma | j^\nu(\bar{x}', 0, 0) | \mathbf{0}, \sigma_0 \rangle}{\omega + E_{\sigma}(\mathbf{K}) - E_{\sigma_0}(\mathbf{0}) - i\delta} \sum_{n''_x} e^{i(k_x + K_x)r_s n''_x} \int_{-\infty}^{\infty} dy'' e^{i(k_y + K_y)y''} \right. \\
&\quad \left. - \frac{\langle \mathbf{0}, \sigma_0 | j^\nu(\bar{x}', 0, 0) | \mathbf{K}, \sigma \rangle \langle \mathbf{K}, \sigma | j^\mu(\bar{x}, 0, 0) | \mathbf{0}, \sigma_0 \rangle}{\omega - E_{\sigma}(\mathbf{K}) + E_{\sigma_0}(\mathbf{0}) + i\delta} \sum_{n''_x} e^{i(k_x - K_x)r_s n''_x} \int_{-\infty}^{\infty} dy'' e^{i(k_y - K_y)y''} \right\}, \tag{3.4}
\end{aligned}$$

where  $y'' = y - y'$  and  $n''_x = n_x - n'_x$ . After carrying out  $y', y''$  integrations and the summations of  $n'_x$  and  $n''_x$ , the delta functions in the correlation functions appear as

$$\begin{aligned}
K_\omega^{\mu\nu}(\mathbf{k}, \mathbf{k}') &= \sum_{\sigma} \mathcal{A} \frac{(2\pi)^2}{r_s^2} \int_{-r_s/2}^{r_s/2} d\bar{x} d\bar{x}' \sum_{N, N'} \int_0^{2\pi/r_s} dK_x \int_{-\infty}^{\infty} dK_y e^{ik_x \bar{x} + ik'_x \bar{x}'} \delta(\mathbf{k} + \mathbf{k}' - \frac{2\pi \mathbf{N}}{r_s}) \\
&\quad \times \left\{ \frac{\langle \mathbf{0}, \sigma_0 | j^\mu(\bar{x}, 0, 0) | \mathbf{K}, \sigma \rangle \langle \mathbf{K}, \sigma | j^\nu(\bar{x}', 0, 0) | \mathbf{0}, \sigma_0 \rangle}{\omega + E_{\sigma}(\mathbf{K}) - E_{\sigma_0}(\mathbf{0}) - i\delta} \delta(\mathbf{k} + \mathbf{K} - \frac{2\pi \mathbf{N}'}{r_s}) \right. \\
&\quad \left. - \frac{\langle \mathbf{0}, \sigma_0 | j^\nu(\bar{x}', 0, 0) | \mathbf{K}, \sigma \rangle \langle \mathbf{K}, \sigma | j^\mu(\bar{x}, 0, 0) | \mathbf{0}, \sigma_0 \rangle}{\omega - E_{\sigma}(\mathbf{K}) + E_{\sigma_0}(\mathbf{0}) + i\delta} \delta(\mathbf{k} - \mathbf{K} - \frac{2\pi \mathbf{N}'}{r_s}) \right\} \\
&= \sum_{\sigma} \mathcal{A} \frac{(2\pi)^2}{r_s^2} \int_{-r_s/2}^{r_s/2} d\bar{x} d\bar{x}' \sum_N \int d^2 K e^{ik_x \bar{x} + ik'_x \bar{x}'} \delta(\mathbf{k} + \mathbf{k}' - \frac{2\pi \mathbf{N}}{r_s}) \\
&\quad \times \left\{ \frac{\langle \mathbf{0}, \sigma_0 | j^\mu(\bar{x}, 0, 0) | \mathbf{K}, \sigma \rangle \langle \mathbf{K}, \sigma | j^\nu(\bar{x}', 0, 0) | \mathbf{0}, \sigma_0 \rangle}{\omega + E_{\sigma}(\mathbf{K}) - E_{\sigma_0}(\mathbf{0}) - i\delta} \delta(\mathbf{K} + \mathbf{k}) \right. \\
&\quad \left. - \frac{\langle \mathbf{0}, \sigma_0 | j^\nu(\bar{x}', 0, 0) | \mathbf{K}, \sigma \rangle \langle \mathbf{K}, \sigma | j^\mu(\bar{x}, 0, 0) | \mathbf{0}, \sigma_0 \rangle}{\omega - E_{\sigma}(\mathbf{K}) + E_{\sigma_0}(\mathbf{0}) + i\delta} \delta(\mathbf{K} - \mathbf{k}) \right\}, \tag{3.5}
\end{aligned}$$

where  $\mathbf{N} = N\mathbf{e}_x$ ,  $\mathbf{N}' = N'\mathbf{e}_x$ ,  $N$  and  $N'$  are integers. In Eq (3.5), we extend the finite  $K_x$  integral region into the infinite region with the use of an assumption  $E_{\sigma}(K_x + 2\pi/r_s, K_y) = E_{\sigma}(\mathbf{K})$ . After the  $\mathbf{K}$  integration, the current correlation function becomes

$$K_\omega^{\mu\nu}(\mathbf{k}, \mathbf{k}') = (2\pi)^2 \sum_N \delta(\hat{\mathbf{k}} + \hat{\mathbf{k}}' - 2\pi \mathbf{N}) \tilde{\Pi}_N^{\mu\nu}(\mathbf{k}, \omega), \tag{3.6}$$

where  $\hat{\mathbf{k}} = (r_s k_x, k_y/r_s)$ .  $\tilde{\Pi}_N^{\mu\nu}(\mathbf{k}, \omega)$  is defined as follows,

$$\begin{aligned}
\tilde{\Pi}_N^{\mu\nu}(\mathbf{k}, \omega) &= \frac{\mathcal{A}}{r_s^2} \sum_{\sigma} \int_{-r_s/2}^{r_s/2} d\bar{x} \int_{-r_s/2}^{r_s/2} d\bar{x}' e^{ik_x \bar{x} + i(k_x - \frac{2\pi N}{r_s}) \bar{x}'} \\
&\quad \times \left\{ \frac{\langle \mathbf{0}, \sigma_0 | j^\mu(\bar{x}, 0, 0) | -\mathbf{k}, \sigma \rangle \langle -\mathbf{k}, \sigma | j^\nu(\bar{x}', 0, 0) | \mathbf{0}, \sigma_0 \rangle}{\omega + E_{\sigma}(-\mathbf{k}) - E_{\sigma_0}(\mathbf{0}) - i\delta} - \frac{\langle \mathbf{0}, \sigma_0 | j^\nu(\bar{x}', 0, 0) | \mathbf{k}, \sigma \rangle \langle \mathbf{k}, \sigma | j^\mu(\bar{x}, 0, 0) | \mathbf{0}, \sigma_0 \rangle}{\omega - E_{\sigma}(\mathbf{k}) + E_{\sigma_0}(\mathbf{0}) + i\delta} \right\}. \tag{3.7}
\end{aligned}$$

We use  $j^\mu(\bar{x}, 0, 0) = \int d^2k' \tilde{j}^\mu(\mathbf{k}', 0) e^{-ik'_x \bar{x}} / (2\pi)^2$  and  $\int_{-r_s/2}^{r_s/2} d\bar{x} e^{i(k_x - k'_x)\bar{x}} = 2 \sin((k_x - k'_x)r_s/2) / (k_x - k'_x)$ . Then we obtain

$$\begin{aligned} \tilde{\Pi}_N^{\mu\nu}(\mathbf{k}, \omega) &= 4\mathcal{A} \sum_{\sigma} \int \frac{d^2k'}{(2\pi)^2} \frac{\sin((\hat{k}_x - \hat{k}'_x)/2)}{\hat{k}_x - \hat{k}'_x} \int \frac{d^2k''}{(2\pi)^2} \frac{\sin((- \hat{k}_x - \hat{k}''_x)/2 + \pi N)}{-\hat{k}_x - \hat{k}''_x + 2\pi N} \\ &\times \left\{ \frac{\langle \mathbf{0}, \sigma_0 | \tilde{j}^\mu(\mathbf{k}', 0) | -\mathbf{k}, \sigma \rangle \langle -\mathbf{k}, \sigma | \tilde{j}^\nu(\mathbf{k}'', 0) | \mathbf{0}, \sigma_0 \rangle}{\omega + E_{\sigma}(-\mathbf{k}) - E_{\sigma_0}(\mathbf{0}) - i\delta} - \frac{\langle \mathbf{0}, \sigma_0 | \tilde{j}^\nu(\mathbf{k}'', 0) | \mathbf{k}, \sigma \rangle \langle \mathbf{k}, \sigma | \tilde{j}^\mu(\mathbf{k}', 0) | \mathbf{0}, \sigma_0 \rangle}{\omega - E_{\sigma}(\mathbf{k}) + E_{\sigma_0}(\mathbf{0}) + i\delta} \right\}. \end{aligned} \quad (3.8)$$

The delta function of  $\mathbf{k}$  and  $\mathbf{k}'$  is periodic in  $k_x$  direction because of the periodicity of the striped Hall gas. When  $\mathbf{k} = 0$ , the cyclotron resonance is derived with the use of Eq. (3.8) in Appendix B. If the magnetic translational symmetries in both  $x$  and  $y$  direction were unbroken, then the delta function would become  $(2\pi)^2 \delta(\mathbf{k} + \mathbf{k}')$  and there would be no trigonometric factor.

## B. Evaluation of the current correlation function

We evaluate the current correlation function of the striped Hall gas in the half-filled  $l$ th LL. The striped Hall gas state spontaneously breaks the magnetic translational symmetry in  $x$  direction and there exists the NG mode. We project the current correlation function onto the  $l$ th LL to study the contribution of the NG mode. Since the species index  $\sigma$  becomes  $\sigma_0$  after the LL projection, we do not write the species index of the LL projected current correlation function in the following in the present paper. We use the UCDW in the HFA<sup>5,6</sup> for the ground state, and use the SMA<sup>14,15,16</sup> for the NG mode in order to evaluate the right hand side of Eq. (3.8).

The UCDW in the HFA and the NG mode in the SMA are discussed by using the von Neumann lattice (vNL) formalism,<sup>18</sup> in which the QHS is represented as a two-dimensional lattice system. In the vNL formalism, the one-electron states are expanded by the vNL basis. Let us introduce the vNL basis. A discrete set of coherent states of guiding-center coordinates,

$$(X + iY) |\alpha_{mn}\rangle = z_{mn} |\alpha_{mn}\rangle, \quad (3.9)$$

is a complete set of the  $(X, Y)$  space. Here we use  $z_{mn} = (mr_s + in/r_s)$  with integers  $m$  and  $n$ . These coherent states are localized at the position  $(mr_s, n/r_s)$ . By Fourier transforming these states, we obtain the orthonormal basis in the momentum representation,  $|\beta_{\mathbf{p}}\rangle = \sum_{mn} e^{ip_x m + ip_y n} |\alpha_{mn}\rangle / \beta(\mathbf{p})$ , where  $\beta(\mathbf{p}) = (2\text{Im}\tau)^{1/4} e^{i\tau p_y^2/4\pi} \vartheta_1((p_x + \tau p_y)/2\pi|\tau)$ . The  $\vartheta_1$  is a Jacobi's theta function and  $\tau = ir_s^2$ . The two-dimensional momentum  $\mathbf{p}$  is defined in the magnetic Brillouin-zone (BZ),  $|p_i| < \pi$ . A discrete set of the eigenstate of the one-particle free Hamiltonian,

$$\frac{m\omega_c^2}{2} (\xi^2 + \eta^2) |f_l\rangle = \omega_c(l + \frac{1}{2}) |f_l\rangle, l = 0, 1, 2, \dots, \quad (3.10)$$

is the complete set of the  $(\xi, \eta)$  space. The Hilbert space is spanned by the direct product of these states,  $|l, \mathbf{p}\rangle = |f_l\rangle \otimes |\beta_{\mathbf{p}}\rangle$ . In the following in this paper, we do not write the time dependence explicitly. A field operator of an electron is expanded by the vNL basis as

$$\Psi(\mathbf{r}) = \sum_{l=0}^{\infty} \int_{\text{BZ}} \frac{d^2p}{(2\pi)^2} b_l(\mathbf{p}) \langle \mathbf{r} | l, \mathbf{p} \rangle, \quad (3.11)$$

where  $b_l(\mathbf{p})$  is the anti-commuting annihilation operator which obeys

$$\{b_l(\mathbf{p}), b_{l'}^\dagger(\mathbf{p}')\} = \delta_{ll'} \sum_N (2\pi)^2 \delta(\mathbf{p} - \mathbf{p}' - 2\pi\mathbf{n}) e^{i\phi(p', n)} \quad (3.12)$$

with a boundary condition  $b_l(\mathbf{p} + 2\pi\mathbf{n}) = e^{i\phi(p, n)} b_l(\mathbf{p})$ . Here,  $\mathbf{n} = (n_x, n_y)$ ,  $n_x$  and  $n_y$  are integers and  $\phi(p, n) = \pi(n_x + n_y) - n_y p_x$ .

The mean-field state of the striped Hall gas in the HFA has an anisotropic Fermi sea (FIG.1) which is uniform in  $p_x$  direction. The mean-field state is constructed as  $|0\rangle = N_1 \prod_{\mathbf{p} \in \text{FS}} b_l^\dagger(\mathbf{p}) |vac\rangle$ . Here  $N_1$  is the normalization constant, FS denotes the Fermi sea and  $|vac\rangle$  is the vacuum state which is occupied up to the  $l-1$ th LL. The two point function is given by

$$\langle 0 | b_l^\dagger(\mathbf{p}) b_{l'}(\mathbf{p}') | 0 \rangle = \delta_{ll'} \theta[\mu - \epsilon_{\text{HF}}^{(l)}(\mathbf{p})] \sum_N (2\pi)^2 \delta(\mathbf{p} - \mathbf{p}' + 2\pi\mathbf{N}) e^{i\phi(p, N)}, \quad (3.13)$$

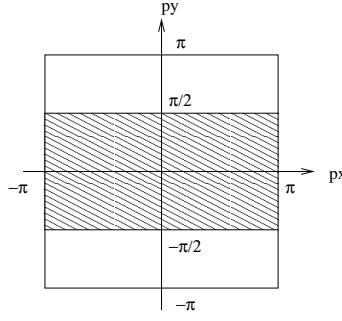


FIG. 1: Fermi sea in the magnetic BZ of  $l$ th Landau level

where  $\mu$  is a chemical potential and  $\epsilon_{\text{HF}}^{(l)}(\mathbf{p})$  is a one-electron energy in the  $l$ th LL. The HF state  $|0\rangle$  is an eigenstate of magnetic translation operators with an eigenvalue  $\mathbf{K} = \mathbf{0}$ .

The SMA is a variational method to calculate the lowest excited state, and is consistent with the GRPA in numerical calculations.<sup>13</sup> The low energy excitation in the SMA is a fluctuation of the density in the striped Hall gas. Given the ground state  $|0\rangle$ , the NG mode in the SMA at  $\mathbf{k}$  is assumed as

$$|\mathbf{k}\rangle = \frac{\tilde{\rho}_*(\mathbf{k})}{\sqrt{N_e^* s(\mathbf{k})}} |0\rangle \quad (3.14)$$

with the projected density operator,

$$\tilde{\rho}_*(\mathbf{k}) = P_l \int d^2r \Psi^\dagger(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{X}} \Psi(\mathbf{r}) P_l = \int_{\text{BZ}} \frac{d^2p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) b_l(\mathbf{p} - \hat{\mathbf{k}}) e^{-i\frac{\hat{k}_x}{4\pi}(2p_y - \hat{k}_y)}, \quad (3.15)$$

where  $P_l$  is a projection operator onto the subspace of the  $l$ th LL.  $N_e^*$  is the electron number in the  $l$ th LL, and the function  $s(\mathbf{k})$ , defined as  $s(\mathbf{k}) = \langle 0 | \tilde{\rho}_*(-\mathbf{k}) \tilde{\rho}_*(\mathbf{k}) | 0 \rangle / N_e^*$ , is the static structure function. The excitation energy in the SMA is given by  $E_{\text{SMA}}(\mathbf{k}) = f(\mathbf{k})/s(\mathbf{k})$ , where  $f(\mathbf{k}) = \langle 0 | [\tilde{\rho}_*(-\mathbf{k}), [H^{(l)}, \tilde{\rho}_*(\mathbf{k})]] | 0 \rangle / 2N_e^*$ . The operator  $H^{(l)} = \int d^2k \tilde{\rho}_*(\mathbf{k}) v_l(k) \tilde{\rho}_*(\mathbf{k}) / (2(2\pi)^2)$  is the Hamiltonian projected onto the  $l$ th LL, where  $v_l(\mathbf{k}) = \exp(-\mathbf{k}^2/4\pi) [L_l(\mathbf{k}^2/4\pi)]^2 2\pi q^2/k$ , and  $L_l(x)$  is the Laguerre polynomial. From our previous paper,<sup>14</sup> we know  $E_{\text{SMA}}(\mathbf{k}) = q^2 |\hat{k}_y| \{ A_0 \hat{k}_x^2 + B_0 \hat{k}_y^4 + O(\hat{k}_x^2 \hat{k}_y^2, \hat{k}_x^4) \}$  and that the NG mode in the SMA is an eigenstate of magnetic translation operators with an eigenvalue  $\mathbf{K} = \mathbf{k}$ .

In terms of  $|0\rangle$  and  $|\mathbf{k}\rangle$ , the matrix element in the integrand of Eq. (3.8), i.e.,  $\langle \mathbf{0}, \sigma_0 | j^\mu(\mathbf{k}') | \mathbf{k}, \sigma \rangle$ , is replaced by

$$\langle 0 | P_l j^\mu(\mathbf{k}') P_l | \mathbf{k} \rangle = \langle 0 | \alpha^\mu(\mathbf{k}') \tilde{\rho}_*(\mathbf{k}') | \mathbf{k} \rangle = \frac{\alpha^\mu(\mathbf{k}')}{\sqrt{N_e^* s(\mathbf{k})}} \langle 0 | \tilde{\rho}_*(\mathbf{k}') \tilde{\rho}_*(\mathbf{k}) | 0 \rangle. \quad (3.16)$$

Here,  $\alpha^\mu(\mathbf{k}')$  is defined by

$$\alpha^\mu(\mathbf{k}') = \begin{cases} e^{-\frac{\mathbf{k}'^2}{8\pi}} L_l(\frac{\mathbf{k}'^2}{4\pi}) & ; \mu = 0 \\ -\sum_{j=x,y} i\omega_c \epsilon^{ij} \frac{\partial}{\partial k'_j} \{ e^{-\frac{\mathbf{k}'^2}{8\pi}} L_l(\frac{\mathbf{k}'}{4\pi}) \} = -\sum_{j=x,y} i\omega_c e^{-\frac{\mathbf{k}'^2}{8\pi}} \epsilon^{ij} \frac{k'_j}{2\pi} \left\{ L'_l(\frac{\mathbf{k}'}{4\pi}) - \frac{1}{2} L_l(\frac{\mathbf{k}'}{4\pi}) \right\} & ; \mu = i. \end{cases} \quad (3.17)$$

From Eqs. (3.12), (3.13) and (3.15), the expectation value of  $\tilde{\rho}_*(\mathbf{k}') \tilde{\rho}_*(\mathbf{k})$  is obtained as follows,

$$\begin{aligned} \langle 0 | \tilde{\rho}_*(\mathbf{k}') \tilde{\rho}_*(\mathbf{k}) | 0 \rangle &= \int_{\text{BZ}} \frac{d^2p}{(2\pi)^2} \int_{\text{BZ}} \frac{d^2p'}{(2\pi)^2} e^{-i\frac{\hat{k}_x}{4\pi}(2p_y - \hat{k}'_y) - i\frac{\hat{k}_x}{4\pi}(2p'_y - \hat{k}_y)} \langle 0 | b_l^\dagger(\mathbf{p}) b_l(\mathbf{p} - \hat{\mathbf{k}}') b_l^\dagger(\mathbf{p}') b_l(\mathbf{p}' - \hat{\mathbf{k}}) | 0 \rangle \\ &= \int_{\text{BZ}} \frac{d^2p}{(2\pi)^2} \int_{\text{BZ}} \frac{d^2p'}{(2\pi)^2} e^{-i\frac{\hat{k}_x}{4\pi}(2p_y - \hat{k}'_y) - i\frac{\hat{k}_x}{4\pi}(2p'_y - \hat{k}_y)} \{ -\langle 0 | b_l^\dagger(\mathbf{p}) b_l^\dagger(\mathbf{p}') b_l(\mathbf{p} - \hat{\mathbf{k}}') b_l(\mathbf{p}' - \hat{\mathbf{k}}) | 0 \rangle \\ &\quad + \sum_{\mathbf{n}} (2\pi)^2 \delta(\mathbf{p} + \mathbf{k}' - \mathbf{p}' - 2\pi\mathbf{n}) e^{i\phi(p', n)} \langle 0 | b_l^\dagger(\mathbf{p}) b_l(\mathbf{p}' - \hat{\mathbf{k}}) | 0 \rangle \} \\ &= \langle \tilde{\rho}_*(\mathbf{k}') \rangle \langle \tilde{\rho}_*(\mathbf{k}) \rangle + e^{i\frac{\hat{k}_x \hat{k}'_y}{4\pi} - i\frac{\hat{k}_x \hat{k}_y}{4\pi}} \sum_{\mathbf{n}} (2\pi)^2 \delta(\hat{\mathbf{k}} + \mathbf{k}' - 2\pi\mathbf{n}) \gamma(n_x, k_y) e^{i\pi n_x} \delta_{n_y, 0}. \end{aligned} \quad (3.18)$$

Here  $\langle 0 | b_l^\dagger(\mathbf{p}) b_l^\dagger(\mathbf{p}') b_l(\mathbf{p} - \hat{\mathbf{k}}') b_l(\mathbf{p}' - \hat{\mathbf{k}}) | 0 \rangle$  is decomposed into the direct term and the exchange term, and the product of  $\langle \tilde{\rho}_*(\mathbf{k}') \rangle$  comes from the direct term. The  $\gamma(n_x, k_y)$  is defined as

$$\gamma(n_x, k_y) = \delta_{n_x 0} \frac{|\hat{k}_y|}{2\pi} + \frac{1}{n_x \pi} \left\{ \sin\left(\frac{n_x \pi}{2}\right) - e^{i \frac{n_x \hat{k}_y}{2}} \sin\left(\frac{n_x}{2}(\pi - |\hat{k}_y|)\right) \right\} (1 - \delta_{n_x 0}). \quad (3.19)$$

The expectation value  $\langle \tilde{\rho}_*(\mathbf{k}) \rangle = \langle 0 | \tilde{\rho}_*(\mathbf{k}) | 0 \rangle = (2\pi)^2 \sum_{n_x} \delta(\hat{k}_x + 2\pi n_x) \delta(\hat{k}_y) \exp(i\pi n_x) \sin(n_x \pi/2)/(n_x \pi)$  vanishes in the small  $\mathbf{k}$  region ( $\mathbf{k} \neq 0$ ). Thus the matrix element in the numerator in Eq. (3.8) in the small  $\mathbf{k}$  region becomes

$$\begin{aligned} \langle 0 | P_l j^\mu(\mathbf{k}') P_l | \mathbf{k} \rangle \langle \mathbf{k} | P_l j^\nu(\mathbf{k}'') P_l | 0 \rangle &= \frac{\alpha^\mu(\mathbf{k}') \alpha^\nu(\mathbf{k}'')}{N_e^* s(\mathbf{k})} e^{i \frac{(\hat{k}'_x \hat{k}'_y - \hat{k}''_x \hat{k}''_y)}{4\pi}} \sum_{n_x, \mathbf{n}'} \gamma(n_x, k_y) \gamma(n'_x, k_y) e^{i\pi(n_x + n'_x)} \delta_{n_y 0} \delta_{n'_y 0} \\ &\times (2\pi)^2 \delta(\hat{\mathbf{k}} + \hat{\mathbf{k}}' - 2\pi \mathbf{n}) (2\pi)^2 \delta(-\hat{\mathbf{k}} + \hat{\mathbf{k}}'' - 2\pi \mathbf{n}'). \end{aligned} \quad (3.20)$$

By inserting Eq. (3.20) into Eq. (3.8) and carrying out the  $\mathbf{k}', \mathbf{k}''$  integrations, the current correlation function  $\tilde{\Pi}_{N*}^{\mu\nu}(\mathbf{k}, \omega)$  in the  $l$ th LL is obtained as

$$\begin{aligned} &\sum_{n_x, n'_x} \frac{(-1)^N \mathcal{A} \alpha^\mu(-k_x + 2\pi n_x/r_s, -k_y) \alpha^\nu(k_x + 2\pi n'_x/r_s, k_y) \gamma(n_x, k_y) \gamma(n'_x, k_y) \sin^2 \hat{k}_x}{N_e^* s(\mathbf{k})(\hat{k}_x - \pi n_x)(\hat{k}_x + \pi n'_x - \pi N)} \\ &\times \left\{ \frac{e^{-i(n_x + n'_x)\hat{k}_y/2}}{\omega + E_{\text{SMA}}(-\mathbf{k}) - i\delta} - \frac{e^{i(n_x + n'_x)\hat{k}_y/2}}{\omega - E_{\text{SMA}}(\mathbf{k}) + i\delta} \right\}. \end{aligned} \quad (3.21)$$

The dominant contribution in the current correlation function  $\tilde{\Pi}_{0*}^{\mu\nu}(\mathbf{k}, \omega)$  comes from the  $n_x = n'_x = 0$  term at the small  $\mathbf{k}$  region due to the factors  $(a\hat{k}_x - \pi n_x)^{-1}$  and  $(a\hat{k}_x + \pi n'_x - \pi N)^{-1}$ . The dominant term of  $\tilde{\Pi}_{0*}^{\mu\nu}(\mathbf{k}, \omega)$  reads

$$\begin{aligned} &\mathcal{A} \left( \frac{|\hat{k}_y|}{2\pi} \frac{\sin \hat{k}_x}{\hat{k}_x} \right)^2 \frac{\alpha^\mu(-\mathbf{k}) \alpha^\nu(\mathbf{k})}{N_e^* s(\mathbf{k})} \left\{ \frac{1}{\omega + E_{\text{SMA}}(-\mathbf{k}) - i\delta} - \frac{1}{\omega - E_{\text{SMA}}(\mathbf{k}) + i\delta} \right\} \\ &= \alpha^\mu(-\mathbf{k}) \alpha^\nu(\mathbf{k}) \frac{|\hat{k}_y|}{2\pi} \left\{ \frac{1}{\omega + E_{\text{SMA}}(-\mathbf{k}) - i\delta} - \frac{1}{\omega - E_{\text{SMA}}(\mathbf{k}) + i\delta} \right\}. \end{aligned} \quad (3.22)$$

Here we use the static structure factor  $s(\mathbf{k}) = |\hat{k}_y|/(2\pi \nu_*)$  at a small  $\mathbf{k}$ , where  $\nu_* = N_e^*/\mathcal{A}$  is a filling factor of the  $l$ th LL. We also use  $\lim_{x \rightarrow 0} \sin x/x = 1$ .

#### IV. IMPLICATIONS OF THE NG MODE

From the current correlation function of the striped Hall gas, we see implications of the NG mode. In the subsection A, we find a sharp energy absorption at the NG mode frequency besides at the cyclotron frequency. In the subsection B, we study the dominant resonance of the density correlation function in the long wavelength limit and show that the density correlation function satisfies the f-sum rule.

##### A. Photon energy absorption rate

When an external electromagnetic wave is added, the striped Hall gas absorbs the photon energy. We study the photon energy absorption rate which is proportional to the current correlation function, and inversely proportional to the frequency  $\omega$  (see appendix A). A sharp absorption occurs at  $\omega = \omega_c$  when  $\mathbf{k} = 0$  (see appendix B). The striped Hall gas has the NG mode  $\omega = \omega_{NG}(\mathbf{k})$ , and a sharp absorption may also occur at the NG mode frequency which vanishes in the  $|\mathbf{k}| \rightarrow 0$  limit.

As a typical example, we assume that the electromagnetic wave is propagating in the three-dimensional space  $(\mathbf{r}, z)$  as  $\mathbf{E} = \mathcal{E}_L(\mathbf{e}_x + i\mathbf{e}_y) e^{i\mathbf{k} \cdot \mathbf{r} + ik_z z - i\omega t}$ , where  $\mathcal{E}_L(\mathbf{e}_x + i\mathbf{e}_y)$  is a left-handed circularly polarization vector, and  $k_z$  is a wave number vector in  $z$  direction. The photon energy absorption rate of the striped Hall gas is expressed by the current correlation function as

$$P = \frac{2e^2 \mathcal{E}_L^2}{\omega} (\text{Im}_{\omega > 0} \tilde{\Pi}_0^{xx}(\mathbf{k}, \omega) + \text{Im}_{\omega > 0} \tilde{\Pi}_0^{yy}(\mathbf{k}, \omega)), \quad (4.1)$$

which is derived in the appendix A.

In order to evaluate the contribution of the NG mode, we project the photon energy absorption rate onto the  $l$ th LL as

$$P_{\text{NG}} = \frac{2e^2\mathcal{E}_L^2}{\omega} (\text{Im}_{\omega>0} \tilde{\Pi}_{0*}^{xx}(\mathbf{k}, \omega) + \text{Im}_{\omega>0} \tilde{\Pi}_{0*}^{yy}(\mathbf{k}, \omega)). \quad (4.2)$$

By inserting Eq. (3.22) into the above equation, we compute the photon energy absorption rate as

$$\begin{aligned} P_{\text{NG}} &= \frac{e^2\mathcal{E}_L^2}{\omega} \sum_{i=x,y} \alpha^i(-\mathbf{k}) \alpha^i(\mathbf{k}) \frac{|\hat{k}_y|}{2\pi} 2\pi \delta(\omega - E_{\text{SMA}}(\mathbf{k})) \\ &= \left( \frac{e\mathcal{E}_L\omega_c(l+\frac{1}{2})}{2\pi} \right)^2 \frac{k_x^2 + k_y^2}{q^2(A_0\hat{k}_x^2 + B_0\hat{k}_y^4)} \delta(\omega - q^2|\hat{k}_y|(A_0\hat{k}_x^2 + B_0\hat{k}_y^4)), \end{aligned} \quad (4.3)$$

where we use  $E_{\text{SMA}}(\mathbf{k}) = q^2|\hat{k}_y|(A_0\hat{k}_x^2 + B_0\hat{k}_y^4)$ , and  $\alpha^i(\mathbf{k}) = i\omega_c(l+1/2)\epsilon^{ij}k_j/(2\pi)$  in the small  $\mathbf{k}$  region. We also used  $e^{-x} = 1$ ,  $L_l(x) = 1$ ,  $L'_l(x) = -l$  in  $x \rightarrow 0$  limit. The ratio  $(k_x^2 + k_y^2)/(A_0\hat{k}_x^2 + B_0\hat{k}_y^4)$  converges to a constant value when  $k_x$  and  $k_y$  approach zero in the same order or when  $k_y$  approaches zero in the higher order than  $k_x$ . In particular when we fix  $k_x$  and take a  $k_y \rightarrow 0$ , the photon energy absorption rate is obtained as follows,

$$P_{\text{NG}} = \left( \frac{e\mathcal{E}_L\omega_c(l+\frac{1}{2})}{2\pi} \right)^2 (q^2 A_0 r_s^2)^{-1} \delta(\omega). \quad (4.4)$$

This result shows a sharp energy absorption at the NG mode frequency. The same result is obtained for the right-handed circularly polarized electromagnetic wave. On the other hand, a sharp energy absorption at  $\omega = \omega_c$  with  $\mathbf{k} = \mathbf{0}$  is derived by using the current correlation function without the LL projection. The result becomes the following (see appendix B);

$$P_{\text{cyclotron}} = \frac{(e\mathcal{E}_L)^2 \rho_e}{m} 2\pi \delta(\omega - \omega_c), \quad (4.5)$$

where  $\rho_e = N_e/\mathcal{A}$ . The cyclotron resonance is not affected by the NG mode. A sharp absorption at  $\omega = E_{\text{SMA}}(\mathbf{k})$  occurs at a small  $\mathbf{k}$  in addition to the sharp absorption at the cyclotron frequency. For the right-handed circularly polarized electromagnetic wave,  $P_{\text{cyclotron}} = 0$ .

The strength of resonant peaks depends on a magnetic field  $B$ . Fig. 2 shows the  $B$ -dependence of the photon energy absorption rate.  $P_{\text{NG}}$  and  $P_{\text{cyclotron}}$  of the striped Hall gas depend on  $B$  through the cyclotron frequency  $\omega_c$  and  $a = \sqrt{2\pi/eB}$  which has been set 1 so far. The photon energy absorption rate at the NG mode frequency in the SI units is  $\lim_{E_{\text{SMA}} \rightarrow 0} P_{\text{NG}}/(\mathcal{E}_L^2 \delta(\omega - E_{\text{SMA}})) = 1.810 \times 10^9 B^{3/2} [\text{s}^2 \text{ A}^2/\text{kg m}]$  with  $l = 2$ ,  $r_s = 2.474$  and  $A_0 = 0.351$ . The value of  $r_s$  is determined by minimizing the HF energy per unit area. The photon energy absorption rate at the cyclotron frequency in the SI units is  $P_{\text{cyclotron}}/(\mathcal{E}_L^2 \delta(\omega - \omega_c)) = 1.572 \times 10^9 B [\text{s}^2 \text{ A}^2/\text{kg m}]$  with  $l = 2$ . The photon energy absorption rate at the NG mode frequency is larger than the photon energy absorption rate at the cyclotron frequency.

The strength of  $P_{\text{NG}}$  depends on the incident angle of the electromagnetic wave. When the electromagnetic wave is tilted parallel to the stripes, the photon energy absorption rate diverges in the  $|\mathbf{k}| \rightarrow 0$  limit. When the electromagnetic wave is tilted perpendicular to the stripes, the photon energy absorption rate becomes finite. The anisotropy of  $P_{\text{NG}}$  is a new property of the striped Hall gas.

## B. Properties of the density correlation function

In this subsection, we discuss properties of the density correlation function which is a temporal diagonal element of the current correlation function in the QHS.

When excitations have finite energy gaps, the dominant resonance of the density correlation function  $K_\omega^{00}(\mathbf{k}, -\mathbf{k})$  occurs at  $\omega = \omega_c$  in the long wavelength limit. Since the striped Hall gas has a zero-energy excitation, there is also a resonance of the density correlation function at the NG mode frequency. We study the NG mode resonance of the density correlation function in the long wavelength limit and find that the dominant resonance occurs at the NG mode frequency.

First we derive the cyclotron resonance in the density correlation function when there is a finite energy gap. The ground state and an excited state are denoted by  $|0\rangle$  and  $|m\rangle$ , respectively. The current conservation law gives

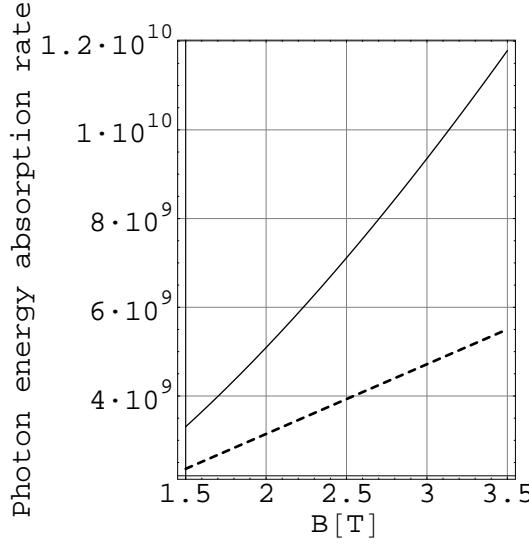


FIG. 2:  $B$ -dependence of the photon energy absorption rate with  $r_s = 2.474$  at  $l = 2$ . The photon absorption rate  $P_{\text{NG}}/(\mathcal{E}_L^2 \delta(\omega - E_{\text{SMA}}))$  at the small  $\mathbf{k}$  is shown by a solid curve, and  $P_{\text{cyclotron}}/(\mathcal{E}_L^2 \delta(\omega - \omega_c))$  is shown by a dashed curve. We fix a filling factor as  $\nu = 2.5$  and change a magnetic field  $B[\text{T}]$  around  $2.5[\text{T}]$  where an anisotropic resistivity at  $\nu = 2.5$  is observed. The unit of the vertical axis is  $[\text{s}^2 \text{ A}^2/\text{kg m}^2]$ .

$\langle m | [H, \tilde{j}^0(\mathbf{k})] | 0 \rangle = i \sum_{i=x,y} k_i \langle m | \tilde{j}^i(\mathbf{k}) | 0 \rangle$ , where  $\tilde{j}^\mu(\mathbf{k}) = \int d^2r j^\mu(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}$ . Thus a matrix element of the density operator  $\langle m | \tilde{j}^0(\mathbf{k}) | 0 \rangle$  is represented by  $i \sum_{i=x,y} k_i \langle 0 | \tilde{j}^i(\mathbf{k}) | m \rangle / (E_m - E_0)$ . By using this matrix element and  $\tilde{j}^i(\mathbf{k}) = \pi^i/m + O(\mathbf{k})$ , we can rewrite the density correlation function as follows,

$$\begin{aligned}
 K_\omega^{00}(\mathbf{k}, -\mathbf{k}) &= \sum_m \left\{ \frac{\langle 0 | \tilde{j}^0(\mathbf{k}) | m \rangle \langle m | \tilde{j}^0(-\mathbf{k}) | 0 \rangle}{\omega + E_m - E_0 - i\delta} - \frac{\langle 0 | \tilde{j}^0(-\mathbf{k}) | m \rangle \langle m | \tilde{j}^0(\mathbf{k}) | 0 \rangle}{\omega - E_m + E_0 + i\delta} \right\} \\
 &= \frac{1}{m^2} \sum_{i,j=x,y} \sum_m \frac{k_i k_j}{(E_m - E_0)^2} \left\{ \frac{\langle 0 | \pi^i | m \rangle \langle m | \pi^j | 0 \rangle}{\omega + E_m - E_0 - i\delta} - \frac{\langle 0 | \pi^j | m \rangle \langle m | \pi^i | 0 \rangle}{\omega - E_m + E_0 + i\delta} + O(\mathbf{k}) \right\} \\
 &= \frac{N_e \mathbf{k}^2}{2\pi} \left\{ \frac{1}{\omega + \omega_c - i\delta} - \frac{1}{\omega - \omega_c + i\delta} \right\} + \frac{1}{m^2} \sum_m \frac{1}{(E_m - E_0)^2} O(\mathbf{k}^3),
 \end{aligned} \tag{4.6}$$

where the operator  $\pi^i$  is the covariant momentum defined as

$$\begin{aligned}
 \pi^x &= - \int d^2r \Psi^\dagger(\mathbf{r}) 2\pi\eta \Psi(\mathbf{r}), \\
 \pi^y &= \int d^2r \Psi^\dagger(\mathbf{r}) 2\pi\xi \Psi(\mathbf{r}).
 \end{aligned} \tag{4.7}$$

When there is a finite energy gap, the  $\mathbf{k}$ -dependence of the energy difference in the second term is negligible. Thus the largest residue of the density correlation function  $O(\mathbf{k}^2)$  appears at the cyclotron frequency, and the residues of the density correlation function at the other excitations are  $O(\mathbf{k}^3)$ . The cyclotron resonance is dominant at a small  $\mathbf{k}$ .<sup>19</sup>

Next we discuss a resonance in the density correlation function when there is a gapless excitation. In this case, we cannot neglect the  $\mathbf{k}$ -dependence of the energy difference. Since the energy difference  $E_m - E_0$  approaches zero as  $\mathbf{k}$  approaches zero and cancels  $k_i$  in the numerator of Eq. (4.6), residues of the density correlation function at gapless excitations in the second term of Eq. (4.6) may become larger than  $O(\mathbf{k}^2)$ . We use the LL projected density correlation function of the striped Hall gas obtained in the previous section, and study a residue of the density correlation function

at the NG mode frequency. Inserting Eq. (3.22) into Eq. (3.6), we obtain the long wavelength limit of the density correlation function in the  $l$ th LL as

$$\begin{aligned} K_{\omega*}^{00}(\mathbf{k}, -\mathbf{k}) &= (2\pi)^2 \delta(0)^2 \tilde{\Pi}_{0*}^{00}(\mathbf{k}, \omega) \\ &= \mathcal{A} \frac{|\hat{k}_y|}{2\pi} \left\{ \frac{1}{\omega + E_{\text{SMA}}(\mathbf{k}) - i\delta} - \frac{1}{\omega - E_{\text{SMA}}(\mathbf{k}) + i\delta} \right\}, \end{aligned} \quad (4.8)$$

where we use  $\alpha^0(\mathbf{k}) = 1$  in the small  $\mathbf{k}$  region. By comparing Eq. (4.6) with Eq. (4.8), we know that the residue of the density correlation function at the NG mode frequency is  $\mathcal{A}|\hat{k}_y|/(2\pi)$ , and the residue of the density correlation function at the cyclotron frequency is  $N_e \mathbf{k}^2/2\pi$ . The density correlation function of the striped Hall gas has the largest residue at the NG mode frequency in the long wavelength limit. Hence, the dominant resonance of the density correlation function occurs at the NG mode frequency.

The  $\tilde{\Pi}_{0*}^{00}(\mathbf{k}, \omega)$  should satisfy the f-sum rule<sup>20</sup> in the subspace of the  $l$ th LL as the following,

$$\int_0^\infty d\omega \omega \text{Im} \tilde{\Pi}_{0*}^{00}(\mathbf{k}, \omega) = \pi \nu_* f(\mathbf{k}). \quad (4.9)$$

The left hand side is calculated in the SMA by using Eq. (3.22), and is given by

$$\int_0^\infty d\omega E_{\text{SMA}}(\mathbf{k}) \frac{|\hat{k}_y|}{2\pi} \pi \delta(\omega - E_{\text{SMA}}(\mathbf{k})). \quad (4.10)$$

Using  $E_{\text{SMA}}(\mathbf{k}) = f(\mathbf{k})/s(\mathbf{k})$  and  $s(\mathbf{k}) = |\hat{k}_y|/(2\pi\nu_*)$ , this becomes  $\pi\nu_* f(\mathbf{k})$  which is the right hand side of Eq. (4.9). Hence,  $\tilde{\Pi}_{0*}^{00}(\mathbf{k}, \omega)$  in the SMA satisfies the f-sum rule. This suggests that the SMA is reasonable.

## V. SUMMARY AND DISCUSSION

In this paper, we have studied the contribution of the NG mode at  $\omega = \omega_{\text{NG}}(\mathbf{k})$  to the current correlation function of the striped Hall gas. The striped Hall gas is the UCDW state which is one of the HFA solutions and has an anisotropic Fermi surface. The striped Hall gas spontaneously breaks the magnetic translational symmetry and the magnetic rotational symmetry. The symmetry breaking causes a gapless NG mode.

We evaluated the current correlation function in the subspace of  $l$ th LL and clarified the  $\mathbf{k}$ -dependence of the LL projected current correlation function. Then, we found that the dominant resonance of the density correlation function occurs not at  $\omega = \omega_c$  but at  $\omega = \omega_{\text{NG}}(\mathbf{k})$  at the small  $\mathbf{k}$ . The density correlation function satisfies f-sum rule in the subspace of  $l$ th LL, then the SMA is reasonable. Using the current correlation function, we saw that the new highly anisotropic photon energy absorption occurs at the NG mode frequency. The photon energy absorption rate at  $\omega = \omega_{\text{NG}}(\mathbf{k})$  depends on the incident direction of the electromagnetic wave. A finite energy absorption occurs when the incident direction is perpendicular to the stripes. When the incident direction is parallel to the stripes, the absorbed energy diverges in the  $|\mathbf{k}| \rightarrow 0$  limit.

When the incident direction of the electromagnetic wave is perpendicular to the stripes, the photon energy absorption rate at  $\omega = \omega_{\text{NG}}(\mathbf{k})$  is proportional to  $B^{3/2}$  and is larger than the photon absorption rate at  $\omega = \omega_c$ . At  $B = 2.5[\text{T}]$  where the anisotropic resistivity at  $\nu = 2.5$  is observed, the striped Hall gas in the half-filled third LL absorbs the photon energy at the NG mode frequency which is twice as large as the absorbed energy at the cyclotron frequency.

When there is a finite excitation energy gap, the sharp absorption only occurs at the cyclotron frequency and is not affected by the electron interactions due to the Kohn's theorem. Without the LL projection, we found a sharp photon energy absorption at  $\omega = \omega_c$  in the striped Hall gas. The photon energy absorption rate at  $\omega = \omega_c$  does not depend on the incident direction of the electromagnetic wave in the  $|\mathbf{k}| \rightarrow 0$  limit. The cyclotron resonance is not affected in the striped Hall gas. The result supplements the Kohn's theorem in the system of the NG mode.

We used the SMA in the present paper. The SMA is one of two approaches to study the low energy excitation. The other approach is based on the edge picture in which the low energy excitations are assumed as small displacements of the edges of the stripes. At a small  $\mathbf{k}$ , the excitation energy in the SMA is proportional to  $|\hat{k}_y|(A_0 \hat{k}_x^2 + B_0 \hat{k}_y^4)$  where  $A_0$  and  $B_0$  are constants. The excitation energy based on the edge picture is proportional to  $|k_y| \{(Y k_x^2 + K k_y^4)/|\mathbf{k}|\}^{1/2}$  where  $Y$  and  $K$  are the compression and the bending elastic moduli<sup>8,9</sup>, respectively. The SMA excitation energy is smaller than the excitation energy based on the edge picture. Hence the SMA is appropriate for the study of the NG mode, and we applied the SMA in this paper.

In summary, We found that the anisotropic photon energy absorption occurs at  $\omega = \omega_{NG}(\mathbf{k})$  in the small  $\mathbf{k}$  region. The photon energy absorption rate at  $\omega = \omega_{NG}(\mathbf{k})$  depends on the incident direction of the electromagnetic wave, whereas the photon energy absorption rate at  $\omega = \omega_c$  does not depend on the direction in the  $|\mathbf{k}| \rightarrow 0$  limit. The absorbed energy at  $\omega = \omega_{NG}(\mathbf{k})$  is larger than the absorbed energy at  $\omega = \omega_c$  when the incident direction is perpendicular to the stripes. We hope that the anisotropic photon energy absorption will be observed in experiments for the evidence of spontaneous breaking of the magnetic translational symmetry in the striped Hall gas.

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### APPENDIX A: DEFINITION OF A PHOTON ENERGY ABSORPTION RATE

In this section, we derive a photon energy absorption rate of the QHS by using the first order perturbation when an electromagnetic wave is added. It is assumed that the electromagnetic wave  $\mathbf{E} = \mathcal{E}e^{i\mathbf{k}_{3D} \cdot \mathbf{r}_{3D} - i\omega t}$  is propagating with the wave number vector  $\mathbf{k}_{3D} = (\mathbf{k}, k_z)$  and oscillating with the frequency  $\omega$  in the spacetime  $(\mathbf{r}_{3D}, t) = (\mathbf{r}, z, t)$ . Then the interaction Hamiltonian  $V_{int}(t) = e \int d^3r \mathbf{A}_{ext}(\mathbf{r}_{3D}) \cdot \mathbf{j}_{3D}(\mathbf{r}_{3D})$  is written with  $\mathbf{A}_{ext} = \mathcal{E}e^{i\mathbf{k}_{3D} \cdot \mathbf{r}_{3D} - i\omega t}/i\omega$  as  $V_{int}(t) = e \int d^3r \mathcal{E} \cdot \mathbf{j}_{3D}(\mathbf{r}_{3D})e^{i\mathbf{k}_{3D} \cdot \mathbf{r}_{3D} - i\omega t}/i\omega = e\mathcal{E} \cdot \tilde{\mathbf{j}}_{3D}(\mathbf{k}_{3D})e^{-i\omega t}/i\omega$ . Here we use a three-dimensional current operator  $\mathbf{j}_{3D} = (j^1(\mathbf{r}_{3D}), j^2(\mathbf{r}_{3D}), j^3(\mathbf{r}_{3D}))$  with  $j^\mu(\mathbf{r}_{3D}) = j^\mu(\mathbf{r})\delta(z)$  for  $\mu = 0, 1, 2$ , and  $j^3(\mathbf{r}_{3D}) = 0$ . Then the Fourier transformation gives  $\tilde{j}^\mu(\mathbf{k}_{3D}) = j^\mu(\mathbf{k})$  and  $\tilde{j}^3(\mathbf{k}_{3D}) = 0$ . By summing up all possible final states, a transition probability from an initial state  $|i\rangle$  to a final state  $|f\rangle$  is given by using the first order perturbation as follows,

$$\begin{aligned} Prob &= \sum_f \left| -i \int_{-T/2}^{T/2} dt \langle f | V_{int}(t) | i \rangle e^{i(E_f - E_i)t} \right|^2 \\ &= e^2 \sum_{l,m=1,2} \frac{\mathcal{E}_l \mathcal{E}_m}{\omega^2} \sum_f F_T(\omega)^2 \langle i | \tilde{j}^l(-\mathbf{k}) | f \rangle \langle f | \tilde{j}^m(\mathbf{k}) | i \rangle. \end{aligned} \quad (A1)$$

Here  $F_T(\omega) = \int_{-T/2}^{T/2} dt e^{-i(\omega - E_f + E_i)t}$ , which satisfies  $\lim_{T \rightarrow \infty} F_T(\omega) = 2\pi\delta(\omega - E_f + E_i)$ . The transition probability per unit time is written as

$$\frac{Prob}{T} = \sum_{l,m=1,2} \sum_f 2\pi\delta(\omega - E_f + E_i) \frac{e^2 \mathcal{E}_l^* \mathcal{E}_m}{\omega^2} \langle i | \tilde{j}^l(-\mathbf{k}) | f \rangle \langle f | \tilde{j}^m(\mathbf{k}) | i \rangle. \quad (A2)$$

Suppose that the initial state  $|i\rangle$  is the ground state  $|0\rangle$ . Then Eq. (A2) is represented by the current correlation function as

$$\frac{Prob}{T} = \sum_{l,m=1,2} \frac{2e^2 \mathcal{E}_l^* \mathcal{E}_m \mathcal{A}}{\omega^2} \text{Im}_{\omega > 0} \tilde{\Pi}_0^{lm}(\mathbf{k}, \omega). \quad (A3)$$

We define the photon energy absorption rate  $P$  as

$$\omega \frac{Prob}{T \cdot \mathcal{A}} = \sum_{l,m=1,2} \frac{2e^2 \mathcal{E}_l^* \mathcal{E}_m}{\omega} \text{Im}_{\omega > 0} \tilde{\Pi}_0^{lm}(\mathbf{k}, \omega). \quad (A4)$$

Suppose that the electromagnetic wave is the left-handed circularly polarized wave with  $\mathcal{E} = \mathcal{E}_L(\mathbf{e}_x + i\mathbf{e}_y)$ , then the photon energy absorption rate is given by

$$P = \frac{2e^2 \mathcal{E}_L^2}{\omega} (\text{Im}_{\omega > 0} \tilde{\Pi}_0^{xx}(\mathbf{k}, \omega) + \text{Im}_{\omega > 0} \tilde{\Pi}_0^{yy}(\mathbf{k}, \omega)). \quad (A5)$$

## APPENDIX B: CYCLOTRON RESONANCE

In this section, we derive the cyclotron resonance when a homogeneous electromagnetic wave is added to the striped Hall gas. The covariant momentum is written as

$$\pi^i = m\tilde{j}^i(\mathbf{0}) \quad (\text{B1})$$

with  $\tilde{j}^i(\mathbf{k}) = \int d^2r \Psi^\dagger(\mathbf{r}) v^i \Psi(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}$ , where  $i = x, y$  and  $v^i = \omega_c(-\eta, \xi)$ . The covariant momentum operators satisfy  $[\pi^x, \pi^y] = -i2\pi Q = -i2\pi N_e$ , where we replace the conserved charge  $Q$  by  $N_e$ . The covariant momentum operators also satisfy the following relation  $i[H, \pi^i] = -2\pi \sum_{j=x,y} \epsilon_{ij} \pi^j / m$ , where  $\epsilon_{ij}$  is antisymmetric tensor and suffix  $i, j$  run  $x, y$ . From these relations, we now define the energy ladder operators  $\pi_\pm$  as

$$\pi_\pm = \frac{1}{\sqrt{4\pi N_e}} (\pi^x \pm i\pi^y). \quad (\text{B2})$$

The energy ladder operators satisfy  $[\pi_-, \pi_+] = 1$ . Then we find that

$$[H, \pi_\pm] = \pm\omega_c \pi_\pm. \quad (\text{B3})$$

Thus,  $\pi_\pm$  shifts the energy of the system by the cyclotron energy  $\omega_c$ . This property indicates that an excited state with the energy  $E_0 + n\omega_c$  exists, where  $H|\mathbf{0}\rangle = E_0|\mathbf{0}\rangle$  and  $n$  is an integer. In the following discussion, we assume that  $\pi_-|\mathbf{0}\rangle = 0$  and  $\langle\mathbf{0}|\pi_+ = 0$ , because of the stability of the ground state. This assumption prohibits the transfer to lower Landau levels than the ground state.

Next, we calculate the photon energy absorption rate in Eq. (A5) with  $\mathbf{k} = 0$ . By using Eq. (3.8),  $\text{Im}_{\omega>0} \tilde{\Pi}_0^{ij}(\mathbf{0}, \omega)$  is written as

$$\sum_{\sigma} 4\mathcal{A} \int \frac{d^2k'}{(2\pi)^2} \frac{d^2k''}{(2\pi)^2} \frac{\sin(\hat{k}'_x/2)}{\hat{k}'_x} \frac{\sin(\hat{k}''_x/2)}{\hat{k}''_x} \langle \mathbf{0}, \sigma_0 | \tilde{j}^i(\mathbf{k}') | \mathbf{0}, \sigma \rangle \langle \mathbf{0}, \sigma | \tilde{j}^j(\mathbf{k}'') | \mathbf{0}, \sigma_0 \rangle \pi \delta(\omega - E_{\sigma}(\mathbf{0}) - E_{\sigma_0}(\mathbf{0})). \quad (\text{B4})$$

We evaluate  $\langle \mathbf{0}, \sigma_0 | \tilde{j}^i(\mathbf{k}') | \mathbf{0}, \sigma \rangle$  where  $|\mathbf{0}, \tau\rangle = \pi_+|\mathbf{0}, \tau_0\rangle$  and  $|\mathbf{0}, \sigma_0\rangle$  is the ground state in the HFA. The current operator is written in the vNL basis as

$$\tilde{j}^i(\mathbf{k}) = \int_{\text{BZ}} \frac{d^2q}{(2\pi)^2} \sum_{l, l'} b_l^\dagger(\mathbf{q}) b_{l'}(\mathbf{q} + \hat{\mathbf{k}}) \langle f_l | \frac{1}{2} \{v^i, e^{-ik\cdot\xi}\} | f_{l'} \rangle e^{\frac{i\hat{k}_x}{4\pi}(2q_y + \hat{k}_y)}. \quad (\text{B5})$$

Taking into account the forbidden transition between occupied states or between empty states, the matrix element in Eq. (B4) is given by

$$\langle \mathbf{0}, \sigma_0 | \tilde{j}^i(\mathbf{k}') | \mathbf{0}, \sigma \rangle = \frac{1}{m\mathcal{A}} \langle \mathbf{0}, \sigma_0 | \pi^i | \mathbf{0}, \sigma \rangle (2\pi)^2 \delta(\mathbf{k}') + \sum_{\mathbf{n} \neq \mathbf{0}} a_{\mathbf{n}, \sigma}^i (2\pi)^2 \delta(\mathbf{k}' + 2\pi\mathbf{n}). \quad (\text{B6})$$

Inserting the matrix element into Eq. (B4), we obtain

$$\sum_{i=x,y} \text{Im}_{\omega>0} \tilde{\Pi}_0^{ii}(\mathbf{0}, \omega) = \frac{1}{\mathcal{A}m^2} \sum_{i=x,y} \sum_{\sigma} \langle \mathbf{0}, \sigma_0 | \pi^i | \mathbf{0}, \sigma \rangle \langle \mathbf{0}, \sigma | \pi^i | \mathbf{0}, \sigma_0 \rangle \pi \delta(\omega - E_{\sigma}(\mathbf{0}) - E_{\sigma_0}(\mathbf{0})) = \frac{2\pi\rho_e}{m^2} \pi \delta(\omega - \omega_c) \quad (\text{B7})$$

where  $\rho_e = N_e/\mathcal{A}$ .  $a_{\mathbf{n}, \sigma}^i (\mathbf{n} \neq \mathbf{0})$  does not contribute to the current correlation function. By inserting this current correlation function into Eq. (A5) with  $\mathbf{k} = \mathbf{0}$ , a sharp energy absorption is obtained as follows,

$$P_{\text{cyclotron}} = \frac{e^2 \mathcal{E}_L^2 \rho_e}{m} 2\pi \delta(\omega - \omega_c). \quad (\text{B8})$$

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